

# Note on Structure Formation from Cosmic String Wakes

Francis Duplessis<sup>1,\*</sup> and Robert Brandenberger<sup>1,†</sup>

<sup>1</sup>*Physics Department, McGill University, 3600 University Street, Montreal, QC, H3A 2T8, Canada*

The search for cosmic strings has been of renewed interest with the advent of precision cosmology. In this note we give a quantitative description of the nonlinear matter density fluctuations that can form from a scaling network of cosmic string wakes. Specifically, we compute the distribution of dark matter halos. These halos would possess strong correlations in position space that should have survived until today. We also discuss the challenges involved in their detection due to their small size and the complex dynamics of their formation.

PACS numbers:

## I. INTRODUCTION

Cosmic strings are linear topological defects predicted by many models of particle physics that go beyond the Standard Model, e.g. in supergravity [1], brane inflation [2] and "String Gas Cosmology" [3] models. In models which admit these defects, a network of cosmic strings is inevitably [4] formed during a phase transition in the very early universe and will persist to the present time. This is true as long as the phase transition occurred after the period of inflation (provided there was inflation). Since strings carry energy, they lead to gravitational effects which induce distinctive signatures in cosmological observations. Since the string tension (which equals the mass per unit length  $\mu$ ) increases as the energy scale  $\eta$  at the time of string formation grows, the magnitude of the predicted signatures of strings increases as the energy scale grows. Thus, a search for observational signatures of strings allows us to constrain the physics occurring at energies much larger than can be probed by earth-based particle accelerators.

It has long been known that cosmic strings are able to produce many interesting effects for cosmology. In particular, cosmic strings lead to a scale-invariant spectrum of cosmological perturbations [5]. Initially, the focus of interest was on strings which have large enough tension to explain the entire amplitude of the power spectrum [5]. Such strings would need to have a value of the tension given (in dimensionless units) by  $G\mu \sim 10^{-6}$ , where  $G$  is Newton's gravitational constant. The perturbations produced by cosmic strings are, however, active and incoherent [6] and hence do not lead to acoustic oscillations in the angular power spectrum of cosmic microwave (CMB) anisotropies. Thus, when conclusive evidence for the existence of these oscillations was reached [7], it became clear that cosmic strings could not be the dominant source of structure formation [8] and interest in cosmological effects of strings collapsed. However, in light of the fact that in many models of inflation (or other explana-

tions for the dominant Gaussian component of the power spectrum of cosmological perturbations) cosmic strings are predicted, there has lately been a revival of interest in searching for cosmic strings (see e.g. [9] for a recent survey). A second reason for the revival of interest in searching for strings is that there have been great advances and technological breakthroughs in observational cosmology which are now making it possible to search for signals of cosmic strings with much lower tensions than those previously considered.

The searches have been progressing in many directions. First of all, cosmic strings will contribute to the power spectrum of density fluctuations [5] and to associated CMB anisotropies [10]. Signatures of these fluctuations can be searched for in optical and infrared galaxy surveys and in CMB temperature anisotropy maps. Cosmic strings leave behind distinctive signals in 21cm redshift surveys [11] and in CMB polarization maps [12]. Considerable work has been done to study the effects of strings in the angular power spectrum of CMB temperature and polarization maps [13]. Analyses combining the SPT [14] and WMAP 7-year [15] data were able to place the bound  $G\mu \leq 1.7 \times 10^{-7}$  [16]<sup>1</sup>, and a similar bound can be obtained [18] using data from the ACT telescope [19]. Improved bounds might be achievable by analyzing CMB maps using statistical tools which are designed to pick out the string-induced non-Gaussianities (see e.g. [20, 21] for some recent studies).

In addition to these purely gravitational effects, cosmic strings can be responsible for the production of highly energetic bursts of particles [22], electromagnetic radiation in a wide range of frequencies [23] and they can help seed coherent magnetic fields on galactic scales [24]. Cosmic string loops decay by emitting gravitational radiation [25]. Hence, strings lead to a scale-invariant stochastic background of gravitational waves [26] which can be constrained by pulsar timing measurements. Cusps on string loops may lead to additional emission of gravitational waves [27], and a resulting constraint on the cosmic string tension of  $G\mu \leq 4 \times 10^{-9}$  [28] has been reported.

\*francis.duplessis@mail.mcgill.ca

†rhh@physics.mcgill.ca

<sup>1</sup> See [17] for earlier bounds using only the WMAP data.

However the bound is sensitive to details of cosmic string cusps which are subject to potentially large back-reaction effects [29]. A bound of  $G\mu \leq 5.2 \times 10^{-7}$  resulting from constraints on the amplitude of gravitational radiation from pulsar timing constraints is more realistic [30].

Observational optical astronomy is also experiencing a phase of rapid progress. Larger telescopes are probing the universe at increasing depth, i.e. at increasing redshift. Since cosmic strings produce non-linear density perturbations at arbitrarily large redshifts, it is expected that the signals of cosmic strings will stand out from the structures produced by the main source of fluctuations (Gaussian fluctuations) more clearly at high redshifts than at low ones. Signals from structures seeded by string loops has been studied in a recent paper by Schlaer et al. [31] (see also [32] for earlier work) who found that loops could cause significant star formation to occur at high redshift. Therefore cosmic strings with enough mass will have an impact on the epoch of reionization [33, 34]. String loops would also seed dense dark matter clumps whose population today could produce a background of gamma radiation sensitive to the Fermi telescope [35].

In this note we address early structure formation from cosmic string wakes. Wakes lead to cosmic structures with a distinctive shape in position space maps. These non-Gaussian features may allow the signals of strings to be detected in a background of Gaussian noise even if the amplitude of the string-induced signal is quite low, as studied for CMB anisotropy maps in [20]. In Section II we give some background about the formation and properties of cosmic string wakes. In Section III we investigate the contribution of string wakes to structure formation as a function of redshift. Finally we discuss the implications and observational signals in Section IV.

## II. BACKGROUND

According to the Kibble mechanism [4], the formation of cosmic strings from a symmetry breaking phase transition implies the existence of a string network. This network is expected to evolve towards a scaling solution which states that its properties become constant if all lengths are scaled to the Hubble radius [36]. The network is composed of two parts. The first is a network of “infinite” strings (which includes loops with radius greater than the Hubble radius) whose curvature radius  $R_c$  can be shown to be of the order of the Hubble scale, i.e.  $R_c = \gamma t$  with  $\gamma \sim \mathcal{O}(1)$ . The fact that the curvature radius scales with  $t$  is maintained by intersections and self-intersections of the long strings. Such intersections create string loops that detach from the long string network, thus allowing the latter to straighten out. This brings us to the second part of the network: a distribution of string loops with curvature radius much smaller than the Hubble scale. Following [8] we model a long string as a superposition of straight string segments, each of

whose length is given by  $R_c$ . This is a reasonable model since any wiggles on the strings would either be redshifted away or would cause self-intersections. The scaling solution suggests  $\tilde{N} \sim 1$  long string per Hubble volume, which is supported by numerical simulations which yield  $\tilde{N} \approx 1 - 10$  with a loop distribution possessing a scaling peak at the loop radius of size  $\approx 1/20$  of the Hubble radius [37] (for earlier work on cosmic string simulations see e.g. [38]).

The geometry of space around a long straight string will be conical with deficit angle  $\alpha = 8\pi G\mu$  [39]. A long string moving with speed  $v_s$  will sweep a plane on which residing static observers would see that matter acquires a velocity kick  $v = 4\pi G\mu v_s \gamma_s$  towards the plane (where  $\gamma_s$  is the relativistic gamma factor associated with the velocity  $v_s$ ). The streams of matter will overlap and create a wedge with twice the background density [40]. The overdense region will eventually collapse under gravitational instability and form a virialized planar structure called the string wake. The growth of the wake will proceed by matter accretion. We will briefly review the dynamics of this process.

Let us consider <sup>2</sup> the effect of the wake on a particle’s motion by labelling its physical distance to the wake by  $r = a(t)(x + \psi(t))$ , where  $x$  is the comoving position,  $\psi(t)$  the displacement and  $a(t)$  the scale factor. The initial conditions are given by  $\psi(t_i) = 0$  and  $\dot{\psi}(t_i) = -\text{sgn}(x)v$ . The Zel’dovich approximation [42] gives as equation of motion,

$$\ddot{\psi} + \frac{4}{3t}\dot{\psi} - \frac{2}{3t^2}\psi = 0. \quad (1)$$

This system can be solved to obtain,

$$\psi(t) = -\frac{12}{5}G\mu\pi v_s \gamma_s t_i \left(\frac{t}{t_i}\right)^{2/3} + \frac{12}{5}G\mu\pi v_s \gamma_s \frac{t_i^2}{t}. \quad (2)$$

After dropping the decaying mode, we can find the distance at which our particle decouples from the Hubble flow through the condition  $\dot{r} = 0$  or more specifically  $x + 2\psi = 0$  which yields,

$$x_{ta}(t) = \pm \frac{24}{5}G\mu\pi v_s \gamma_s t_i \left(\frac{t}{t_i}\right)^{2/3}. \quad (3)$$

Hence  $w(t) = \frac{1}{2}a(t)x_{ta}(t)$ , which denotes the distance from the wake’s center of the mass shell that turns around at  $t$ , is given by

$$w(t) = \frac{12}{5}\pi G\mu v_s \gamma_s t_i \left(\frac{t}{t_i}\right)^{4/3}. \quad (4)$$

Matter that turned around at a height  $w(t_{ta})$  will virialize at a radius  $\frac{1}{2}w(t_{ta})$  showing that  $w(t)$  labels the

---

<sup>2</sup> Such calculations were first performed for string wakes in [41].

width of the wake which contains four times the background density of matter. Shells of infalling baryonic matter will collide with one another at the virial height and create shocks on either side of the wake as shown through hydrodynamical simulations [43, 44]. The energy of the falling particles will be thermalized in the wake giving a temperature [11],

$$\frac{3}{2}k_B T = \frac{1}{2}mv_{shell}^2, \quad (5)$$

where  $v_{shell}$  is the speed of hydrogen particles of mass  $m$  at the shock. A shell that turns around at time  $t_{ta}$  will hit the wake at  $t = (1 + 1/\sqrt{2})^{3/2}t_{ta}$ , and therefore the speed will be given by

$$\begin{aligned} v_{shell} &= \dot{r}(t, t_{ta} = t(1 + 1/\sqrt{2})^{-3/2}) \\ &= \frac{4}{5}(3 - 2\sqrt{2})v\left(\frac{t}{t_i}\right)^{1/3}, \end{aligned} \quad (6)$$

which yields a wake temperature of

$$T = \frac{16}{75}(4(3 - 2\sqrt{2}))^2 \frac{m}{k_B} (G\mu)^2 (v_s \gamma_s)^2 \left(\frac{t}{t_i}\right)^{2/3} \quad (7)$$

$$\simeq [10K](G\mu)_6^2 (v_s \gamma_s)^2 \frac{z_i + 1}{z + 1}, \quad (8)$$

with  $(G\mu)_6$  being the value of  $G\mu$  in units of  $10^{-6}$ .

### III. THE MASS FUNCTION

To compute the amount of matter accreted onto wakes, and eventually the mass function, we must understand how to model the network of wakes created from our scaling solution. As previously mentioned, we expect  $\tilde{N}$  strings per Hubble volume. The curvature radius of the long strings network grows in length relative to fixed comoving coordinates. This growth is realized because long string segments intersect and produce string loops. The string intercommutations occur roughly once per Hubble expansion time per string per Hubble volume. They also lead to a change of the direction of motion of a string after intersecting another string and exchanging ends. We will use a toy model introduced in [8] to characterize the long string network analytically: we divide the time axis into Hubble time steps. In each time step and in each Hubble volume we lay down  $\tilde{N}$  straight string segments of length  $\gamma t$  where  $\gamma$  is less but close to unity, and each string has a velocity  $v_s$  in a random direction. We take the string segments to be uncorrelated. Each string segment lives for a Hubble time. The string network at different Hubble time steps is taken to be uncorrelated.

Wakes first form behind strings at the time of equal matter-radiation energy density  $t_{eq}$ , when matter perturbations can start to grow. We consider Hubble time steps from  $t_{eq}$  to  $t_0$  labelling them  $t_1 = t_{eq}$ ,  $t_2 = 2t_{eq}$ , ...,  $t_m = 2^{m-1}t_{eq}$ . At the beginning of each time step  $t_m$  we create  $\tilde{N}$  string segments, at the end of the time step we annihilate them. Each of these strings will create a wake with

initial dimensions  $\gamma t_m \times v_s \gamma_s t_m \times w(t_m)$  (where  $\gamma_s$  is the relativistic gamma factor associated with the string velocity  $v_s$ ) which will then grow in planar dimensions due to the Hubble expansion and in width via the matter accretion process discussed in the previous section.

Consider a redshift  $z$ , then the comoving number density of wakes laid down at time  $z_m > z$  is given by

$$\tilde{n}_{wake}(z_m) = \frac{\tilde{N}}{H_m^{-3}} \frac{1}{(z_m + 1)^3}. \quad (9)$$

Therefore the mass density in all wakes at a redshift  $z$  is given by summing the contribution from all the wakes created at different Hubble time steps:

$$M_w(z) = 4\rho_b(z) \sum_{z_m > z} \tilde{n}_{wake}(z_m) (1 + z)^3 \text{Vol}_{wake}(z_m, z), \quad (10)$$

where the last factor is the volume at redshift  $z$  of the nonlinear region about a wake created at a redshift  $z_m > z$ . Since the planar dimensions of the wake expand with the Hubble flow, we obtain

$$\begin{aligned} M_w(z) &= 4\rho_b(z) \sum_{z_m > z} \tilde{n}_{wake}(z_m) (1 + z)^3 \\ &\quad \times (\gamma t_m \times v_s \gamma_s t_m \times w(z)) \left(\frac{z_m + 1}{z + 1}\right)^2 \\ &= 4\rho_b(z) \sum_{z_m > z} \frac{32}{45} \pi \tilde{N} G\mu \gamma v_s^2 \gamma_s^2 \frac{1 + z_m}{1 + z}. \end{aligned} \quad (11)$$

In Figure 1 we plot the quantity  $M_w(z)/\rho_b(z)$  (where  $\rho_b(z)$  is the background energy density) for the following parameter choices:

$$G\mu = 1.5 \times 10^{-7}, \quad \tilde{N} = 10, \quad v_s = 1/2 \text{ and } \gamma = 1, \quad (12)$$

where the value of  $G\mu$  is close to the current upper bound and the other values are taken to agree with those in recent cosmic string evolution simulations. This quantity represents the fraction of the total mass in the universe accreted onto wakes.

Figure 1 shows that a considerable fraction of matter is accreted by wakes. Let us analyze this a bit more carefully. Small density perturbations on the surface of the wake will grow due to gravitational instability and might eventually cause the wake to fragment. The dynamics of this process will be quite complex due to the inner structure of the wake: the dark matter density peaks at the edges while baryonic matter clusters in the center [43]. An analysis of the fragmentation was done in the simpler case of a static isothermal sheet-like cloud in [45]. The timescale of the perturbation growth is of the order of the freefall time with the length of the most unstable mode equalling the sheet's width and the longest wavelength unstable mode being twice that. Taking the very optimistic route and assuming our wake behaves in

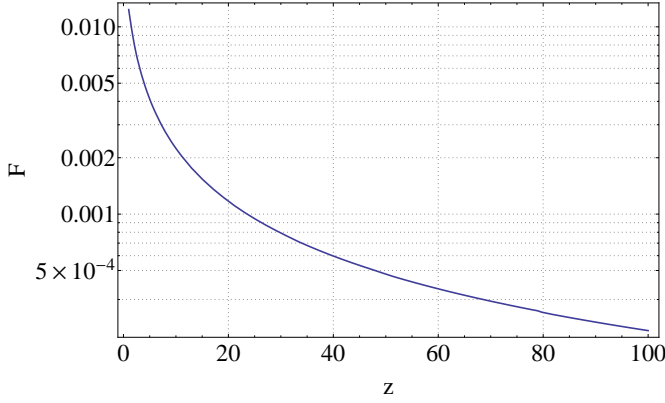


FIG. 1: Fraction of the total matter in the universe that is in cosmic string wakes at various redshifts.

a similar way<sup>3</sup>, long filaments of virial diameter  $\approx w(t)$  should form in a similar fashion to the Zel'dovich pancakes. These filaments would subsequently break into beads themselves, again the fastest growing mode having a length  $\approx 2\pi w(t)$  [31]. Therefore the wakes will eventually fragment into virialized halos of size  $\approx \pi(w/2)^3$ .

With this picture in mind we can compute the comoving number density of halos of mass greater than  $M$  that fragments from the wakes created at redshift  $z_m$ :

$$\begin{aligned} n(z, z_m, > M) &= \tilde{n}_{wake}(z_m) \frac{\text{Vol}_{wake}}{\text{Vol}_{halo}} \Theta(M_{halo} - M) \\ &= \tilde{n}_{wake}(z_m) (\gamma t_m \times v_s \gamma_s t_m \times w(z)) \left( \frac{z_m + 1}{z + 1} \right)^2 \\ &\quad \times \frac{1}{\frac{4}{3}\pi w(z)^3} \Theta(2\rho_b(z) \frac{4\pi}{3} w(z)^3 - M), \end{aligned} \quad (13)$$

where  $\Theta$  is the Heaviside step function.

Taking into account the all wakes that exist at redshift  $z$  we simply get

$$n(z, > M) = \sum_{z_m > z} n(z, z_m, > M). \quad (14)$$

The collection of step functions given by the sum is a bit odd as a distribution since in reality this should be continuous. The discrepancy arises because of the model that considers the wakes to be instantaneously created while this process actually spans a Hubble time. The part of a wake that is created close to time  $t_m$  will have different properties than the part created at

around  $2t_m$  which would be more similar to the beginning of the next set of wakes. Hence we should smear out the distribution  $n(z, > M)$  to have something realistic. To do this we fix  $z$  and split the mass range in intervals  $\Delta M_m = [M(z, z_m), M(z, z_{m-1})]$  with  $M(z, z_j)$  being the mass of the halo's formed at  $z$  from a wake created at the Hubble time step  $z_j$ . Since the values of  $n(z, > M(z_j)) = \sum_{z_m > z_j} n(z, z_m, > M)$  are known, we can just interpolate to create a continuous  $n(z, > M)$ . One can then obtain the comoving mass function from  $\frac{dn(z, > M)}{d \log M}$ .

In Figures 2 and 3, we plot and compare  $n(z, > M)$  and  $\frac{dn(z, > M)}{d \log M}$  from our model to the standard predictions as found in Reed et al. [47] based on Gaussian fluctuations in a standard  $\Lambda$ CDM model with cosmological parameters chosen to agree with those obtained from the WMAP data. We use the string parameters given in Equation 12 and note that the small wiggles in the plots of  $n(z, > M)$  are just an artefact of our interpolation. We see from the figures that structures formed from primordial fluctuations dominate at redshifts smaller than  $z \sim 20$ , whereas the string wakes dominate at higher redshifts. One thing to note is that the mass function for Gaussian perturbations is obtained using the extended Press-Schechter formalism [48] which counts the small halos and their parent halos independently. For wakes, we assume here that smaller halos get destroyed when they merge in the wake to create larger ones, and hence the tail end of the halo distribution could be larger than what we predict here.

Note that even if at some redshift the wake halos make up a decent fraction of the total halo number in the small mass region, objects of these sizes are hard to detect. Therefore it will be challenging to probe this mass range directly with any decent precision.

The statistics studied here do not take into account the distinct position space correlations that wake halos would inherit between each other. The wake-induced halos would be concentrated in sheets, and hence might be visible above a Gaussian noise with significantly larger amplitude using topological statistics such as Minkowski functionals [49]. In the early days of interest in cosmic strings, the application of Minkowski functionals to distinguish structures seeded by strings from that seeded by Gaussian noise was analyzed in [50]. Minkowski functionals have also recently been applied to the analysis of the signals of cosmic strings in 21cm redshift maps [51].

Note, however, that the position space correlations might be disrupted by the interactions between the wakes and the primordial perturbations, something that we have so far neglected. The structures corresponding to density fluctuations on scales larger than the largest wake's width cannot be affected by these interactions. However, the formation of these large objects might destroy any wake in their proximity. We give a rough estimate of the number of surviving wakes by determining the fraction of space that are voids with no large halos.

<sup>3</sup> The neglect of Hubble flow along the wake might not be justified. In an expanding universe structures with planar, cylindrical and spherical symmetry undergo a self-similar growth proportional to  $t^{4/3}$ ,  $t$  and  $t^{8/9}$  respectively [46]. In this case one could argue that if perturbations grew as cylindrical or spherical objects, they would grow slower than the planar wake and therefore no clear fragmentation would occur.

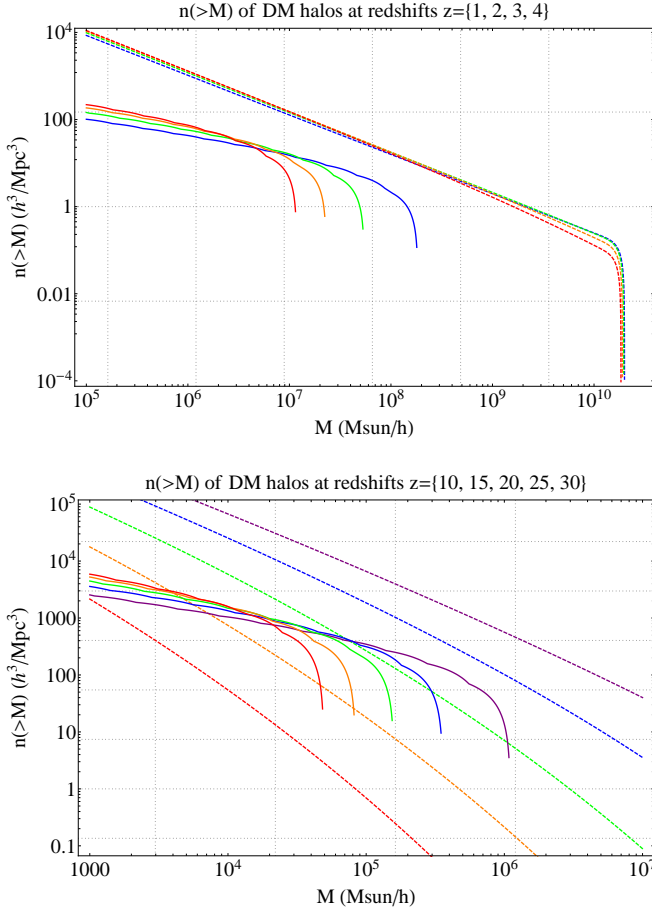


FIG. 2:  $n(z, > M)$  of dark matter halos for a string network with parameters 12 (solid) and the Reed et al. [47] predictions (dashed) at different redshifts. The colors with largest wavelength corresponds to the largest redshifts.

The fraction of space that collapsed in Gaussian noise-induced halos of mass greater than  $M$  is given by [52],

$$F(> M, z) = 2 \int_{\delta_{crit}(z)}^{\infty} d\delta \frac{1}{\sqrt{2\pi}\sigma(M)} \exp\left(-\frac{\delta^2}{2\sigma(M)^2}\right) = \text{Erfc}\left(\frac{\delta_{crit}(z)}{\sqrt{2}\sigma(M)}\right). \quad (15)$$

Here,  $\delta_{crit} = 1.686(1+z)$  and  $\sigma(M)$  is the *rms* density fluctuation on a comoving length scale  $R = (3M/4\pi\rho_b(0))^{1/3}$  which depends on the power spectrum of matter. Figure 4 shows  $F(> M_w, z)$  for three different values of  $G\mu$ . Here  $M_w(z, G\mu)$  is the maximum mass of wake halos. We also plot  $F(> 10^3 M_\odot/h, z)$  which was the smallest mass that was resolved in the simulation by Reed et al. [47]. If  $F$  is close to 1, then most of the matter was accreted onto large Gaussian noise-induced halos, and therefore large halos wiped out any geometrical structures and spatial correlations that wake halos could possess. As the figure shows, the larger the redshift is, the less will be the washout of string-induced structures by the Gaussian noise.

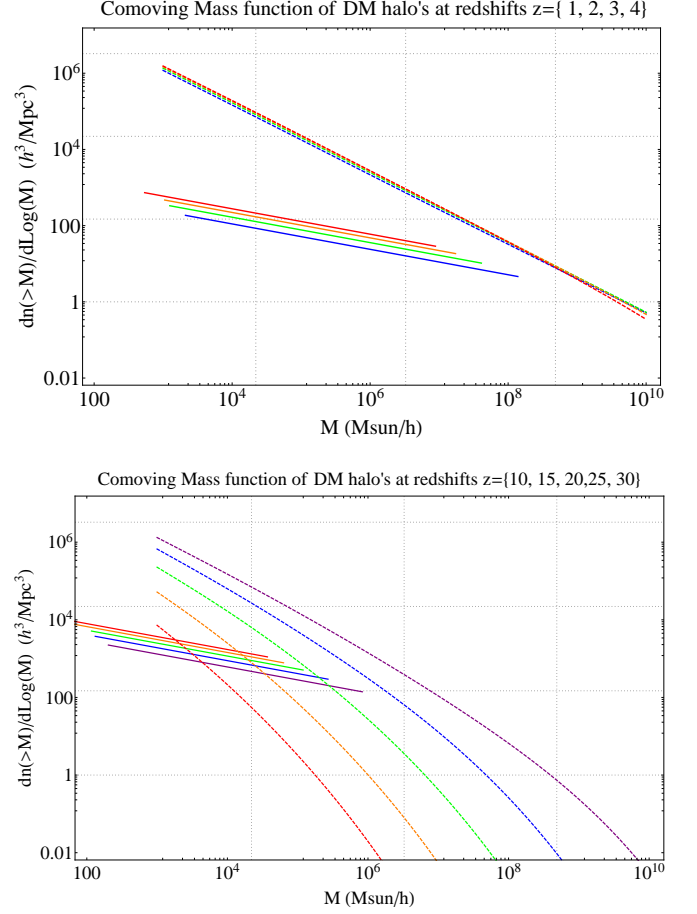


FIG. 3: Mass function of dark matter halos for a string network with parameters 12 (solid) and the Reed et al. [47] predictions (dashed) at different redshifts. The colors with largest wavelength corresponds to the largest redshifts.

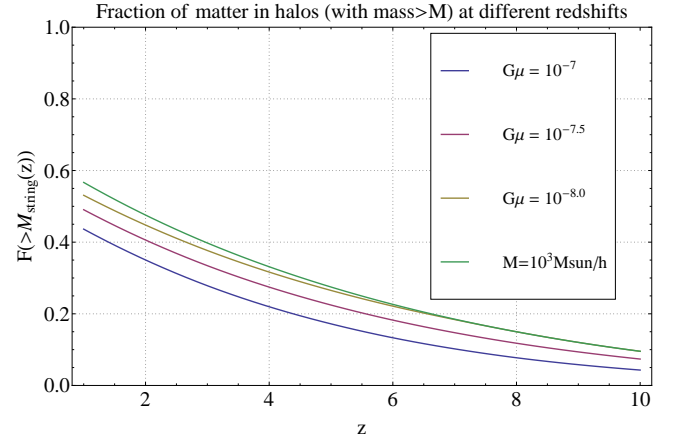


FIG. 4: Fraction of matter in halos of mass greater than wakes can create. We plot the curves for multiple values of  $G\mu$  and one of a mass of  $10^3 M_\odot$  for reference. These were computed with the program supplied in [47].

#### IV. DISCUSSION

In this paper we have studied the formation of halos of dark matter by cosmic string wakes as a function of redshift. We have found that the relative importance of string wakes increases as the redshift increases. However, there are challenges in trying to detect these signals, and this is the topic we discuss in this section without giving too many details.

The temperature of our halos can be estimated from the virial theorem and conservation of energy by considering a collapsing gas cloud of radius  $w$  containing two times the background density  $\rho_b$ ,

$$2\frac{1}{2}v_{\text{virial}}^2 = \frac{3}{5}\frac{GM}{r_{\text{virial}}}, \quad (16)$$

$$\frac{1}{2}v_{\text{virial}}^2 - \frac{3}{5}\frac{GM}{r_{\text{virial}}} = \frac{1}{2}v_{\text{th}}^2 - \frac{3}{5}\frac{GM}{w}. \quad (17)$$

Here,  $w$  is the radius of our collapsing cloud,  $M = 2\rho_b\frac{4}{3}\pi(w)^3$  is its mass and  $v_{\text{th}} = v_{\text{shell}}$  is the initial thermal velocity in Eq. 6. The shock heated baryonic gas will have a temperature of

$$T = \frac{\mu m_p}{2 k_b} v^2, \quad (18)$$

where  $m_H$  is the mass of the proton and  $\mu$  is the mean molecular weight [52]. Star formation can occur if the temperatures are high enough so that colliding molecules can excite each other to higher energy states and radiate away their kinetic energy. Atomic cooling requires  $T > 10^4\text{K}$  while  $H_2$  cooling has channels that can be efficient down to temperatures as low as 200K. However these  $H_2$  molecules are very fragile and get destroyed by the radiation of the first stars, and therefore this cooling mechanism alone will not allow for multiple stars to form in close proximity as would be the case for halos formed from wakes. Figure 5 shows the temperature of the hottest temperatures that can be achieved with the parameters in Eq. 12 and  $\mu \approx 1.22$  in the case of neutral hydrogen. As we can see, the temperatures are far too low to have any significant star formation, and hence no noticeable effect on the history of reionization will be present.

Gravitational lensing could also be an interesting window considering there should be about  $\sim 10^6$  wakes in our Hubble volume. However, even if each wake contains a lot of mass, it is spread over a large area and hence we would only obtain strong lensing if the background source lay, with us, very close to the plane delimited by a wake, in order to have enough mass along the integrated line of sight. This makes the signal too rare to be useful to statistically constrain string parameters. However, if one were lucky, assuming the images do not overlap, which could happen for extended source and a small  $G\mu$  or if we are not in the proximity of the wake's plane, a

lensed source would appear as two images with no magnification or distortion. In such case one could hope to

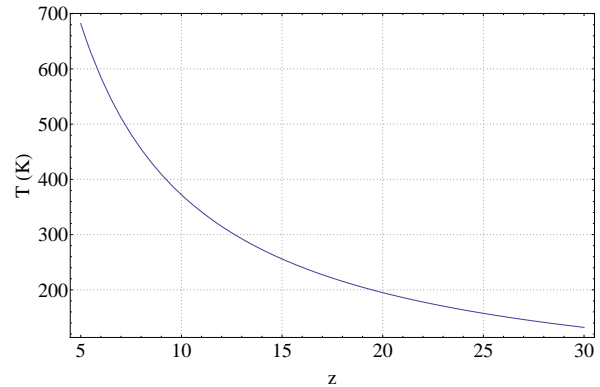


FIG. 5: Redshift dependance of the temperature in the hottest halos that are formed from wakes. We use the parameters in Eq. 12 and  $\mu \approx 1.22$ . Note that in the entire redshift range shown, the halo temperature is higher than both the CMB background temperature and the average kinetic temperature of the baryonic gas.

find other similarly lensed objects along some line in the sky.

Observables sensitive to the structures formed at high redshift would be ideal to differentiate and detect wake structures from the background. One such example, that relies on the compact objects, is gamma ray constraints from dense dark matter clumps [35, 53]. However, the formation of these objects must proceed by a clean radial infall of matter in order to have a reliable density profile to which the gamma ray flux is sensitive. These are not favorable conditions in the wake since we expect strong tidal forces as growing perturbations oscillate around the center.

In conclusion, we have shown that string wakes dominate the nonlinear structures in the universe at sufficiently high redshifts. Even at redshifts where they are not dominant, they may be identifiable because of the distinct spatial correlations which they induce. However, the string-induced halos are typically too small and cold to induce star formation, and hence it is a challenge to be able to find them observationally. One promising window - studied in other works [11, 51, 54–56] - is via 21cm redshift surveys.

#### V. ACKNOWLEDGEMENTS

We would like to thank P. Scott, G. Holder, Y. Omori, Y. Hezaveh, J. Harnois-Deraps and U.L. Pen for useful discussions and comments. This work has been supported in part by an NSERC Discovery Grant and by funds from the Canada Research Chair program.

- 
- [1] R. Jeannerot, “A Supersymmetric SO(10) Model with Inflation and Cosmic Strings,” *Phys. Rev. D* **53**, 5426 (1996) [arXiv:hep-ph/9509365];  
R. Jeannerot, J. Rocher and M. Sakellariadou, “How generic is cosmic string formation in SUSY GUTs,” *Phys. Rev. D* **68**, 103514 (2003) [arXiv:hep-ph/0308134].
- [2] S. Sarangi and S. H. H. Tye, “Cosmic string production towards the end of brane inflation,” *Phys. Lett. B* **536**, 185 (2002) [arXiv:hep-th/0204074];  
E. J. Copeland, R. C. Myers and J. Polchinski, “Cosmic F- and D-strings,” *JHEP* **0406**, 013 (2004) [arXiv:hep-th/0312067].
- [3] R. H. Brandenberger and C. Vafa, “Superstrings in the Early Universe,” *Nucl. Phys. B* **316**, 391 (1989);  
A. Nayeri, R. H. Brandenberger and C. Vafa, “Producing a scale-invariant spectrum of perturbations in a Hagedorn phase of string cosmology,” *Phys. Rev. Lett.* **97**, 021302 (2006) [arXiv:hep-th/0511140];  
R. H. Brandenberger, A. Nayeri, S. P. Patil and C. Vafa, “String gas cosmology and structure formation,” *Int. J. Mod. Phys. A* **22**, 3621 (2007) [hep-th/0608121];  
R. H. Brandenberger, “String Gas Cosmology,” arXiv:0808.0746 [hep-th].
- [4] T. W. B. Kibble, “Topology of Cosmic Domains and Strings,” *J. Phys. A* **9**, 1387 (1976);  
T. W. B. Kibble, “Some Implications of a Cosmological Phase Transition,” *Phys. Rept.* **67**, 183 (1980);  
T. W. B. Kibble, “Phase Transitions In The Early Universe,” *Acta Phys. Polon. B* **13**, 723 (1982).
- [5] Y. B. Zeldovich, “Cosmological fluctuations produced near a singularity,” *Mon. Not. Roy. Astron. Soc.* **192**, 663 (1980);  
A. Vilenkin, “Cosmological Density Fluctuations Produced By Vacuum Strings,” *Phys. Rev. Lett.* **46**, 1169 (1981) [Erratum-ibid. **46**, 1496 (1981)];  
N. Turok and R. H. Brandenberger, “Cosmic Strings And The Formation Of Galaxies And Clusters Of Galaxies,” *Phys. Rev. D* **33**, 2175 (1986);  
H. Sato, “Galaxy Formation by Cosmic Strings,” *Prog. Theor. Phys.* **75**, 1342 (1986);  
A. Stebbins, “Cosmic Strings and Cold Matter”, *Ap. J. (Lett.)* **303**, L21 (1986).
- [6] J. Magueijo, A. Albrecht, D. Coulson and P. Ferreira, “Doppler peaks from active perturbations,” *Phys. Rev. Lett.* **76**, 2617 (1996) [arXiv:astro-ph/9511042];  
U. L. Pen, U. Seljak and N. Turok, “Power spectra in global defect theories of cosmic structure formation,” *Phys. Rev. Lett.* **79**, 1611 (1997) [arXiv:astro-ph/9704165];  
L. Perivolaropoulos, “Spectral Analysis Of Microwave Background Perturbations Induced By Cosmic Strings,” *Astrophys. J.* **451**, 429 (1995) [arXiv:astro-ph/9402024].
- [7] P. D. Mauskopf *et al.* [Boomerang Collaboration], “Measurement of a Peak in the Cosmic Microwave Background Power Spectrum from the North American test flight of BOOMERANG,” *Astrophys. J.* **536**, L59 (2000) [arXiv:astro-ph/9911444].
- [8] L. Perivolaropoulos, “COBE versus cosmic strings: An Analytical model,” *Phys. Lett. B* **298**, 305 (1993) [arXiv:hep-ph/9208247];  
L. Perivolaropoulos, “Statistics of microwave fluctuations induced by topological defects,” *Phys. Rev. D* **48**, 1530 (1993) [arXiv:hep-ph/9212228].
- [9] R. H. Brandenberger, “Topological defects and structure formation,” *Int. J. Mod. Phys. A* **9**, 2117 (1994) [arXiv:astro-ph/9310041].
- [10] J. H. Traschen, N. Turok and R. H. Brandenberger, “Microwave Anisotropies from Cosmic Strings,” *Phys. Rev. D* **34**, 919 (1986);  
R. H. Brandenberger and N. Turok, “Fluctuations From Cosmic Strings And The Microwave Background,” *Phys. Rev. D* **33**, 2182 (1986).
- [11] R. H. Brandenberger, R. J. Danos, O. F. Hernandez and G. P. Holder, “The 21 cm Signature of Cosmic String Wakes,” *JCAP* **1012**, 028 (2010) [arXiv:1006.2514 [astro-ph.CO]].
- [12] R. J. Danos, R. H. Brandenberger and G. Holder, “A Signature of Cosmic Strings Wakes in the CMB Polarization,” *Phys. Rev. D* **82**, 023513 (2010) [arXiv:1003.0905 [astro-ph.CO]].
- [13] L. Pogosian & M. Wyman, *B-modes from Cosmic Strings*, *Phys. Rev. D* **77**, 083509 (2008) [arXiv:0711.0747].  
A. Avgoustidis *et al.*, *Constraints on the fundamental string coupling from B-mode experiments*, *Phys. Rev. Lett.* **107**, 121301 (2011) [arXiv:1105.6198].
- [14] J. E. Ruhl *et al.* [The SPT Collaboration], “The South Pole Telescope,” *Proc. SPIE Int. Soc. Opt. Eng.* **5498**, 11 (2004) [arXiv:astro-ph/0411122];  
J. E. Carlstrom *et al.*, “The 10 Meter South Pole Telescope,” *Publ. Astron. Soc. Pac.* **123**, 568 (2011) [arXiv:0907.4445 [astro-ph.IM]].
- [15] D. Larson, J. Dunkley, G. Hinshaw, E. Komatsu, M. R. Nolta, C. L. Bennett, B. Gold and M. Halpern *et al.*, “Seven-Year Wilkinson Microwave Anisotropy Probe (WMAP) Observations: Power Spectra and WMAP-Derived Parameters,” *Astrophys. J. Suppl.* **192**, 16 (2011) [arXiv:1001.4635 [astro-ph.CO]].
- [16] C. Dvorkin, M. Wyman and W. Hu, “Cosmic String constraints from WMAP and the South Pole Telescope,” *Phys. Rev. D* **84**, 123519 (2011) [arXiv:1109.4947 [astro-ph.CO]].
- [17] L. Pogosian, S. H. H. Tye, I. Wasserman and M. Wyman, “Observational constraints on cosmic string production during brane inflation,” *Phys. Rev. D* **68**, 023506 (2003) [Erratum-ibid. **73**, 089904 (2006)] [arXiv:hep-th/0304188];  
M. Wyman, L. Pogosian and I. Wasserman, “Bounds on cosmic strings from WMAP and SDSS,” *Phys. Rev. D* **72**, 023513 (2005) [Erratum-ibid. **73**, 089905 (2006)] [arXiv:astro-ph/0503364];  
A. A. Fraisse, “Limits on Defects Formation and Hybrid Inflationary Models with Three-Year WMAP Observations,” *JCAP* **0703**, 008 (2007) [arXiv:astro-ph/0603589];  
U. Seljak, A. Slosar and P. McDonald, “Cosmological parameters from combining the Lyman-alpha forest with CMB, galaxy clustering and SN constraints,” *JCAP* **0610**, 014 (2006) [arXiv:astro-ph/0604335];  
R. A. Battye, B. Garbrecht and A. Moss, “Constraints on supersymmetric models of hybrid inflation,” *JCAP* **0609**, 007 (2006) [arXiv:astro-ph/0607339];



- R. A. Battye, B. Garbrecht, A. Moss and H. Stoica, “Constraints on Brane Inflation and Cosmic Strings,” *JCAP* **0801**, 020 (2008) [arXiv:0710.1541 [astro-ph]];
- N. Bevis, M. Hindmarsh, M. Kunz and J. Urrestilla, “CMB power spectrum contribution from cosmic strings using field-evolution simulations of the Abelian Higgs model,” *Phys. Rev. D* **75**, 065015 (2007) [arXiv:astro-ph/0605018];
- N. Bevis, M. Hindmarsh, M. Kunz and J. Urrestilla, “Fitting CMB data with cosmic strings and inflation,” *Phys. Rev. Lett.* **100**, 021301 (2008) [astro-ph/0702223 [ASTRO-PH]];
- R. Battye and A. Moss, “Updated constraints on the cosmic string tension,” *Phys. Rev. D* **82**, 023521 (2010) [arXiv:1005.0479 [astro-ph.CO]].
- [18] J. Urrestilla, N. Bevis, M. Hindmarsh, and M. Kunz, “Cosmic string parameter constraints and model analysis using small scale Cosmic Microwave Background data,” *JCAP* **1112**, 021 (2011) [arXiv:1108.2730 [astro-ph.CO]].
- [19] A. Kosowsky [the ACT Collaboration], “The Atacama Cosmology Telescope Project: A Progress Report,” *New Astron. Rev.* **50**, 969 (2006) [arXiv:astro-ph/0608549].
- [20] S. Amsel, J. Berger and R. H. Brandenberger, “Detecting Cosmic Strings in the CMB with the Canny Algorithm,” *JCAP* **0804**, 015 (2008) [arXiv:0709.0982 [astro-ph]];
- A. Stewart and R. Brandenberger, “Edge Detection, Cosmic Strings and the South Pole Telescope,” *JCAP* **0902**, 009 (2009) [arXiv:0809.0865 [astro-ph]];
- R. J. Danos and R. H. Brandenberger, “Canny Algorithm, Cosmic Strings and the Cosmic Microwave Background,” *Int. J. Mod. Phys. D* **19**, 183 (2010) [arXiv:0811.2004 [astro-ph]].
- [21] M. S. Movahed, B. Javanmardi and R. K. Sheth, “Peak-peak correlations in the cosmic background radiation from cosmic strings,” arXiv:1212.0964 [astro-ph.CO].
- [22] V. Berezhinsky, K. D. Olum, E. Sabancilar and A. Vilenkin, “UHE neutrinos from superconducting cosmic strings,” *Phys. Rev. D* **80**, 023014 (2009) [arXiv:0901.0527 [astro-ph.HE]];
- C. Lunardini and E. Sabancilar, “Cosmic Strings as Emitters of Extremely High Energy Neutrinos,” *Phys. Rev. D* **86**, 085008 (2012) [arXiv:1206.2924 [astro-ph.CO]].
- [23] Y. -F. Cai, E. Sabancilar, D. A. Steer and T. Vachaspati, “Radio Broadcasts from Superconducting Strings,” *Phys. Rev. D* **86**, 043521 (2012) [arXiv:1205.3170 [astro-ph.CO]];
- T. Vachaspati, “Cosmic Sparks from Superconducting Strings,” *Phys. Rev. Lett.* **101**, 141301 (2008) [arXiv:0802.0711 [astro-ph]];
- Y. -F. Cai, E. Sabancilar and T. Vachaspati, “Radio bursts from superconducting strings,” *Phys. Rev. D* **85**, 023530 (2012) [arXiv:1110.1631 [astro-ph.CO]];
- V. Berezhinsky, B. Hnatyk and A. Vilenkin, “Gamma-ray bursts from superconducting cosmic strings,” *Phys. Rev. D* **64**, 043004 (2001) [astro-ph/0102366];
- K. S. Cheng, Y. -W. Yu and T. Harko, “High Redshift Gamma-Ray Bursts: Observational Signatures of Superconducting Cosmic Strings?,” *Phys. Rev. Lett.* **104**, 241102 (2010) [arXiv:1005.3427 [astro-ph.HE]].
- [24] R. H. Brandenberger and X. -m. Zhang, “Anomalous global strings and primordial magnetic fields,” *Phys. Rev. D* **59**, 081301 (1999) [hep-ph/9808306].
- [25] T. Vachaspati and A. Vilenkin, “Gravitational Radiation from Cosmic Strings,” *Phys. Rev. D* **31**, 3052 (1985);
- R. L. Davis, “Nucleosynthesis Problems for String Models of Galaxy Formation,” *Phys. Lett. B* **161**, 285 (1985).
- [26] R. H. Brandenberger, A. Albrecht and N. Turok, “Gravitational Radiation From Cosmic Strings And The Microwave Background,” *Nucl. Phys. B* **277**, 605 (1986).
- [27] T. Damour and A. Vilenkin, “Gravitational radiation from cosmic (super)strings: Bursts, stochastic background, and observational windows,” *Phys. Rev. D* **71**, 063510 (2005) [hep-th/0410222];
- S. Olmez, V. Mandic and X. Siemens, “Gravitational-Wave Stochastic Background from Kinks and Cusps on Cosmic Strings,” *Phys. Rev. D* **81**, 104028 (2010) [arXiv:1004.0890 [astro-ph.CO]].
- [28] R. van Haasteren, Y. Levin, G. H. Janssen, K. Lazaridis, M. K. B. W. Stappers, G. Desvignes, M. B. Purver and A. G. Lyne *et al.*, “Placing limits on the stochastic gravitational-wave background using European Pulsar Timing Array data,” arXiv:1103.0576 [astro-ph.CO].
- [29] R. H. Brandenberger, “On The Decay Of Cosmic String Loops,” *Nucl. Phys. B* **293**, 812 (1987).
- [30] S. A. Sanidas, R. A. Battye and B. W. Stappers, “Constraints on cosmic string tension imposed by the limit on the stochastic gravitational wave background from the European Pulsar Timing Array,” *Phys. Rev. D* **85**, 122003 (2012) [arXiv:1201.2419 [astro-ph.CO]].
- [31] B. Shlaer, A. Vilenkin and A. Loeb, “Early structure formation from cosmic string loops,” *JCAP* **1205**, 026 (2012) [arXiv:1202.1346 [astro-ph.CO]].
- [32] R. Moessner and R. H. Brandenberger, “Formation of high redshift objects in a cosmic string theory with hot dark matter,” *Mon. Not. Roy. Astron. Soc.* **280**, 797 (1996) [astro-ph/9510141].
- [33] H. Tashiro, E. Sabancilar and T. Vachaspati, “Constraints on Superconducting Cosmic Strings from Early Reionization,” *Phys. Rev. D* **85**, 123535 (2012) [arXiv:1204.3643 [astro-ph.CO]].
- [34] K. D. Olum and A. Vilenkin, “Reionization from cosmic string loops,” *Phys. Rev. D* **74**, 063516 (2006) [astro-ph/0605465].
- [35] V. S. Berezhinsky, V. I. Dokuchaev and Y. .N. Eroshenko, “Dense DM clumps seeded by cosmic string loops and DM annihilation,” *JCAP* **1112**, 007 (2011) [arXiv:1107.2751 [astro-ph.HE]].
- [36] T. W. B. Kibble, “Evolution Of A System Of Cosmic Strings,” *Nucl. Phys. B* **252**, 227 (1985) [Erratum-ibid. **B** **261**, 750 (1985)].
- [37] J. J. Blanco-Pillado, K. D. Olum and B. Shlaer, “Large parallel cosmic string simulations: New results on loop production,” *Phys. Rev. D* **83**, 083514 (2011) [arXiv:1101.5173 [astro-ph.CO]].
- [38] A. Albrecht and N. Turok, “Evolution Of Cosmic Strings,” *Phys. Rev. Lett.* **54**, 1868 (1985);
- D. P. Bennett and F. R. Bouchet, “Evidence For A Scaling Solution In Cosmic String Evolution,” *Phys. Rev. Lett.* **60**, 257 (1988);
- B. Allen and E. P. S. Shellard, “Cosmic String Evolution: A Numerical Simulation,” *Phys. Rev. Lett.* **64**, 119 (1990);
- C. Ringeval, M. Sakellariadou and F. Bouchet, “Cosmological evolution of cosmic string loops,” *JCAP* **0702**, 023 (2007) [arXiv:astro-ph/0511646];
- A. A. Fraisse, C. Ringeval, D. N. Spergel and F. R. Bouchet, “Small-Angle CMB Temperature



- Anisotropies Induced by Cosmic Strings,” *Phys. Rev. D* **78**, 043535 (2008) [arXiv:0708.1162 [astro-ph]]; V. Vanchurin, K. D. Olum and A. Vilenkin, “Scaling of cosmic string loops,” *Phys. Rev. D* **74**, 063527 (2006) [arXiv:gr-qc/0511159]; C. Ringeval and F. R. Bouchet, “All sky CMB map from cosmic strings integrated Sachs-Wolfe effect,” arXiv:1204.5041 [astro-ph.CO].
- [39] A. Vilenkin, “Gravitational Field Of Vacuum Domain Walls And Strings,” *Phys. Rev. D* **23**, 852 (1981); R. Gregory, “Gravitational Stability of Local Strings,” *Phys. Rev. Lett.* **59**, 740 (1987).
- [40] J. Silk and A. Vilenkin, “Cosmic Strings And Galaxy Formation,” *Phys. Rev. Lett.* **53**, 1700 (1984); M. Rees, “Baryon concentrations in string wakes at  $z \geq 200$ : implications for galaxy formation and large-scale structure,” *Mon. Not. R. astr. Soc.* **222**, 27p (1986); T. Vachaspati, “Cosmic Strings and the Large-Scale Structure of the Universe,” *Phys. Rev. Lett.* **57**, 1655 (1986); A. Stebbins, S. Veeraraghavan, R. H. Brandenberger, J. Silk and N. Turok, “Cosmic String Wakes,” *Astrophys. J.* **322**, 1 (1987); E. Bertschinger, “Cosmological Accretion Wakes,” *Astrophys. J.* **316**, 489 (1987).
- [41] L. Perivolaropoulos, R. H. Brandenberger and A. Stebbins, “Dissipationless Clustering Of Neutrinos In Cosmic String Induced Wakes,” *Phys. Rev. D* **41**, 1764 (1990); R. H. Brandenberger, L. Perivolaropoulos and A. Stebbins, “Cosmic Strings, Hot Dark Matter and the Large Scale Structure of the Universe,” *Int. J. Mod. Phys. A* **5**, 1633 (1990).
- [42] Y. B. Zeldovich, “Gravitational instability: An Approximate theory for large density perturbations,” *Astron. Astrophys.* **5**, 84 (1970).
- [43] A. Sornborger, R. H. Brandenberger, B. Fryxell and K. Olson, “The Structure of cosmic string wakes,” *Astrophys. J.* **482**, 22 (1997) [astro-ph/9608020].
- [44] T. Hara and S. Miyoshi, “Formation Of The First Systems In The Wakes Of Moving Cosmic Strings,” *Prog. Theor. Phys.* **77**, 1152 (1987).
- [45] S.M. Miyama, S. Narita & C. Hayashi, *Fragmentation of Isothermal Sheet-Like Clouds. I*, *Prog. Theor. Phys.* **78** (1987), 1051.
- [46] J. A. Fillmore and P. Goldreich, “Self-similar gravitational collapse in an expanding universe,” *Astrophys. J.* **281**, 1 (1984).
- [47] D. Reed, R. Bower, C. Frenk, A. Jenkins and T. Theuns, “The halo mass function from the dark ages through the present day,” *Mon. Not. Roy. Astron. Soc.* **374**, 2 (2007) [astro-ph/0607150].
- [48] J. R. Bond, S. Cole, G. Efstathiou and N. Kaiser, “Excursion set mass functions for hierarchical Gaussian fluctuations,” *Astrophys. J.* **379**, 440 (1991).
- [49] K. R. Mecke, T. Buchert, H. Wagner, “Robust morphological measures for large scale structure in the universe,” *Astron. Astrophys.* **288**, 697-704 (1994). [astro-ph/9312028]; J. Schmalzing and T. Buchert, “Beyond genus statistics: a unifying approach to the morphology of cosmic structure,” *Astrophys. J.* **482**, L1 (1997) [arXiv:astro-ph/9702130].
- [50] D. Mitsouras, R. H. Brandenberger and P. Hickson, “Topological Statistics and the LMT Galaxy Redshift Survey,” arXiv:astro-ph/9806360; H. Trac, D. Mitsouras, P. Hickson and R. H. Brandenberger, “Topology of the Las Campanas Redshift Survey,” *Mon. Not. Roy. Astron. Soc.* **330**, 531 (2002) [arXiv:astro-ph/0007125].
- [51] E. McDonough and R. H. Brandenberger, “Searching for Signatures of Cosmic String Wakes in 21cm Redshift Surveys using Minkowski Functionals,” arXiv:1109.2627 [astro-ph.CO].
- [52] R. Barkana and A. Loeb, “In the beginning: The First sources of light and the reionization of the Universe,” *Phys. Rept.* **349**, 125 (2001) [astro-ph/0010468].
- [53] P. Scott and S. Sivertsson, “Gamma-Rays from Ultra-compact Primordial Dark Matter Minihalos,” *Phys. Rev. Lett.* **103**, 211301 (2009) [Erratum-ibid. **105**, 119902 (2010)] [arXiv:0908.4082 [astro-ph.CO]].
- [54] O. F. Hernandez, Y. Wang, R. Brandenberger and J. Fong, “Angular 21 cm Power Spectrum of a Scaling Distribution of Cosmic String Wakes,” *JCAP* **1108**, 014 (2011) [arXiv:1104.3337 [astro-ph.CO]].
- [55] O. F. Hernandez and R. H. Brandenberger, “The 21 cm Signature of Shock Heated and Diffuse Cosmic String Wakes,” *JCAP* **1207**, 032 (2012) [arXiv:1203.2307 [astro-ph.CO]].
- [56] M. Pagano and R. Brandenberger, “The 21cm Signature of a Cosmic String Loop,” *JCAP* **1205**, 014 (2012) [arXiv:1201.5695 [astro-ph.CO]].